Topic: Squeeze theorem

Question: Use squeeze theorem to find the limit.

$$\lim_{x \to 0} x^3 \cos x$$

Answer choices:

A ∞

B -1

C 0

D 1

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Solution: C

To solve this limit we'll need to use the squeeze theorem. In this case we'll create an inequality dealing with just the trigonometric function and then we'll slowly build back up to our original limit.

Taking the trigonometric part of our original limit, and because we know that $\cos x$ will exist only between -1 and 1, we start with the inequality

$$-1 \leq \cos x \leq 1$$

Next we can multiply each term in our inequality by x^3.

$$-x^3 \leq x^3 \cos x \leq x^3$$

We can see that the middle function is our original function. Next, we will take the limits of $-x^3$ and x^3 as $x \to 0$.

The limit of $-x^3$ is

$$\lim_{x \to 0} -x^3$$

Evaluate the limit.

$$\lim_{x \to 0} -x^3 = -(0)^3$$

$$\lim_{x \to 0} -x^3 = 0$$

The limit of x^3 is

$$\lim_{x \to 0} x^3$$

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Evaluate the limit.

$$\lim_{x \to 0} x^3 = (0)^3$$

$$\lim_{x \to 0} x^3 = 0$$

We can put these limit results into our inequality

$$0 \leq x^3 \cos x \leq 0$$

This means that our original function must also equal 0. So based on the squeeze theorem,

$$\lim_{x \to 0} x^3 \cos x = 0$$

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Topic: Squeeze theorem

Question: Use squeeze theorem to find the limit.

$$\lim_{x\to\infty} \frac{\sin(6x)}{x^2}$$

Answer choices:

A -1

B 0

C ∞

D 1

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Solution: B

To solve this limit we'll need to use the squeeze theorem. In this case we'll create an inequality dealing with just the trigonometric function and then we'll slowly build back up to our original limit.

Taking the trigonometric part of our original limit, and because we know that $\sin x$ will exist only between -1 and 1, we start with the inequality

$$-1 \leq \sin(6x) \leq 1$$

Next we can multiply each term in our inequality by $1/x^2$.

$$-\frac{1}{x^2} \leq \frac{\sin(6x)}{x^2} \leq \frac{1}{x^2}$$

Now, we can see that the middle function is our original function. Next we'll take the limits of $-1/x^2$ and $1/x^2$ as $x \to \infty$.

The limit of $-1/x^2$ is

$$\lim_{x\to\infty} -\frac{1}{x^2}$$

Evaluate the limit.

$$\lim_{x\to\infty} -\frac{1}{x^2} = -\frac{1}{\infty}$$

Remember, when we have a fraction with a constant numerator and an increasingly large denominator, the fraction will always approach 0.

$$\lim_{x\to\infty} -\frac{1}{x^2} = 0$$

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The limit of $1/x^2$ is

$$\lim_{x\to\infty} \frac{1}{x^2}$$

Evaluate the limit.

$$\lim_{x\to\infty} \frac{1}{x^2} = \frac{1}{\infty}$$

$$\lim_{x\to\infty} \frac{1}{x^2} = 0$$

We can put these limit results into our inequality

$$0 \leq \frac{\sin(6x)}{x^2} \leq 0$$

This means that our original function must also equal zero. So based on the squeeze theorem

$$\lim_{x\to\infty} \frac{\sin(6x)}{x^2} = 0$$

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Topic: Squeeze theorem

Question: Use squeeze theorem to find the limit.

$$\lim_{x\to\infty} \frac{2x^3 + \sin(4x)}{6x^3 + 5}$$

Answer choices:

A $\frac{1}{3}$

B ∞

C 0

D 3

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Solution: A

To solve this limit we'll need to use the squeeze theorem. In this case we'll create an inequality dealing with just the trigonometric function and then we'll slowly build back up to our original limit.

Taking the trigonometric part of our original limit, and because we know that $\sin x$ will exist only between -1 and 1, we start with the inequality

$$-1 \leq \sin(4x) \leq 1$$

Next we can add $2x^3$ to each term in our inequality.

$$2x^3 - 1 \leq 2x^3 + \sin(4x) \leq 2x^3 + 1$$

Now we can divide each term by $6x^3 + 5$.

$$\frac{2x^3 - 1}{6x^3 + 5} \leq \frac{2x^3 + \sin(4x)}{6x^3 + 5} \leq \frac{2x^3 + 1}{6x^3 + 5}$$

We can see that the middle function is our original function. Next, we will take the limits of

$$\frac{2x^3 - 1}{6x^3 + 5}$$

and

$$\frac{2x^3 + 1}{6x^3 + 5}$$

as $x \to \infty$.

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The limit of $\dfrac{2x^3-1}{6x^3+5}$ is

$$\lim_{x\to\infty}\frac{2x^3-1}{6x^3+5}$$

If we try to evaluate our limit at this point, we'll get ∞/∞, a non-real answer. We'll need to manipulate our limit before we evaluate it. We can start by dividing the numerator and the denominator by the highest degree of the variable, x^3.

$$\lim_{x\to\infty}\frac{2x^3-1}{6x^3+5}=\frac{2x^3-1}{6x^3+5}\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

$$\lim_{x\to\infty}\frac{2x^3-1}{6x^3+5}=\frac{\frac{2x^3}{x^3}-\frac{1}{x^3}}{\frac{6x^3}{x^3}+\frac{5}{x^3}}$$

$$\lim_{x\to\infty}\frac{2x^3-1}{6x^3+5}=\frac{2-\frac{1}{x^3}}{6+\frac{5}{x^3}}$$

Evaluate the limit.

$$\lim_{x\to\infty}\frac{2x^3-1}{6x^3+5}=\frac{2-\frac{1}{\infty}}{6+\frac{5}{\infty}}$$

Remember, when we have a fraction with a constant numerator and an increasingly large denominator, the fraction will always approach 0.

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$$\lim_{x\to\infty}\frac{2x^3-1}{6x^3+5}=\frac{2-0}{6+0}$$

$$\lim_{x\to\infty}\frac{2x^3-1}{6x^3+5}=\frac{1}{3}$$

The limit of $\dfrac{2x^3+1}{6x^3+5}$ is

$$\lim_{x\to\infty}\frac{2x^3+1}{6x^3+5}$$

If we try to evaluate our limit at this point, we'll get ∞/∞, a non-real answer. We'll need to manipulate our limit before we evaluate it. We can start by dividing the numerator and the denominator by the highest degree of the variable, x^3.

$$\lim_{x\to\infty}\frac{2x^3+1}{6x^3+5}=\frac{2x^3+1}{6x^3+5}\left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}}\right)$$

$$\lim_{x\to\infty}\frac{2x^3+1}{6x^3+5}=\frac{\frac{2x^3}{x^3}+\frac{1}{x^3}}{\frac{6x^3}{x^3}+\frac{5}{x^3}}$$

$$\lim_{x\to\infty}\frac{2x^3+1}{6x^3+5}=\frac{2+\frac{1}{x^3}}{6+\frac{5}{x^3}}$$

Evaluate the limit.

$$\lim_{x\to\infty}\frac{2x^3+1}{6x^3+5}=\frac{2+\frac{1}{\infty}}{6+\frac{5}{\infty}}$$

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$$\lim_{x\to\infty}\frac{2x^3+1}{6x^3+5}=\frac{2+0}{6+0}$$

$$\lim_{x\to\infty}\frac{2x^3+1}{6x^3+5}=\frac{1}{3}$$

We can put these limit results into our inequality

$$\frac{1}{3}\leq\frac{2x^3+\sin(4x)}{6x^3+5}\leq\frac{1}{3}$$

This means that our original function must also equal $1/3$. So based on the squeeze theorem

$$\lim_{x\to\infty}\frac{2x^3+\sin(4x)}{6x^3+5}=\frac{1}{3}$$

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